Eliminating first-order internal multiples with downward reflection at the shallowest interface: theory and initial examples

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SUMMARY

The Inverse Scattering Series (ISS) is capable of directly achieving all processing objectives through specific-task subseries and without any subsurface information. In this work a subseries of the ISS is isolated, with the specific task of removing internal multiples of first-order, with downward reflection at the shallowest reflector. The algorithm predicts both the phase and exact amplitude of the internal multiples and does not modify any primary; therefore the internal multiples are removed surgically. This algorithm may be relevant and provide added value when one of the internal multiples under discussion is interfering destructively with (or is proximal to) a primary, and the attenuation of the internal multiple provided by previous algorithms is not adequate for the clean removal of the multiple and not touching the primary. To show how the elimination subseries proposed in this work deals with this challenging situation, an analytic example with three interfaces is included, with one of the relevant first-order internal multiples interfering destructively with the primary generated at the third reflector. We show in particular how the interfering internal multiple is eliminated with no damage to the amplitude or the phase of the primary, as is expected from a method for surgical removal of internal multiples.

INTRODUCTION

Using the ISS and the concept of specific-task subseries, a multidimensional direct algorithm was derived in Araújo (1994), Araújo et al. (1994) and Weglein et al. (1997), to predict and attenuate internal multiples present in the data of a seismic experiment. Prediction methods are followed by the energy-minimization adaptive subtraction to try to accommodate all shortcomings in the prediction. However, there are situations in which the energy-minimization adaptive subtraction technique is not suitable anymore, and the attenuation of internal multiples is not enough for a correct interpretation of the seismic data. An example of this challenging situation for the oil industry can arise when an internal multiple is interfering destructively with (or is proximal to) a primary associated to a target e.g. subsalt targets. This situation is often present in onshore exploration, but it can also happen offshore. While the energy-minimization adaptive subtraction technique is of value for isolated multiples, in this case it might also affect the primary interfering with the internal multiple.

Therefore, it is important to develop new algorithms with enhanced capabilities. In response to this need, Ramírez and Weglein (2005) and Ramírez (2007) discuss early ideas for moving attenuation of internal multiples towards elimination through higher order terms in the ISS. Those ideas and concepts are here progressed and developed leading to a subseries which surgically removes at the same time all internal multiples of first-order having their single downward reflection generated at the shallowest reflector (we will refer to those events as internal multiples generated at the shallowest reflector/interface).

As with any other subseries from the ISS previously isolated, this algorithm requires no subsurface information. We also illustrate how to use this subseries in a three-interface analytic model, to surgically remove the first-order internal multiple generated at the shallowest interface and with both upward reflections generated at the second reflector. The parameters of the model are chosen to allow the internal multiple to interfere destructively with the primary generated at the third reflector.

REVIEW OF THE LEADING-ORDER ATTENUATOR

The Inverse Scattering Series (ISS) is a direct inversion method which can in principle determine, in seismic applications, subsurface properties of the earth using only the measured data D in a seismic experiment, and a Green's function for a chosen reference medium. Unfortunately, with no a priori information of the subsurface of the earth, the convergence is highly restricted (Carvalho 1992).

However, specific-task subseries with different objectives in the chain of data processing can be isolated, and have better convergence properties than the entire ISS. In regard of internal multiples, a subseries was isolated in Araújo (1994) and Weglein et al. (1997), with the specific task of the attenuation of internal multiples of all orders (the order of an internal multiple is defined as the number of downward reflections it experiences anywhere during its travel time. See Figure 1).

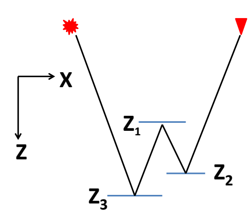

Figure 1: First-order internal multiple: we say, based on the position where reflections occur, that the interfaces generating a first-order internal multiple are in a "lower-higher-lower" configuration.

This Internal Multiple Attenuation Subseries (IMAS) requires that 1) the data D have been deghosted, 2) the reference wave field and free-surface multiples have also been removed from the data and 3) the source wavelet has been deconvolved. The first term of this subseries is the result of the uncollapsed Stolt's migration of the data using the water speed, c_0. The second

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term, conveniently named the *leading-order attenuator*, attenuates all first-order internal multiples at a single step and is of third-order in the measured data.

It turns out that the elimination subseries isolated in this work shares with the IMAS the first two terms, i.e., the data migrated at water speed and the leading-order attenuator. However, both subseries differ from each other for higher-order terms. Hence, we review here the leading-order attenuator, and in the next section we explain how to isolate the higher-order contributions to the elimination subseries.

We will restrict our discussion to a 1D earth with data generated by waves at normal incidence. In this case, the analytic expression for the leading-order attenuator is (Weglein et al. 2003)

$$b_3(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''), \quad (1)$$

where ε is a small and positive parameter introduced to ensure the characteristic “lower-higher-lower” configuration for first-order internal multiples, and to avoid configurations including contributions from the self-interactions, which are defined by the conditions $z'' = z'$ and $z' = z$ in eq. (1). Also, the input $b_1(z)$ of the leading-order attenuator is the first term of the subseries, i.e., the deghosted data migrated at water speed using uncollapsed Stolt's migration. The subindexes in $b_1(k)$ and $b_3(k)$ mean that they are of first-order and third-order respectively in the data.

In the following, we will restrict to the 1D model shown in Figure 2, where Z_i denotes the depth of the i-th. reflector for $i = 1,2,3$.

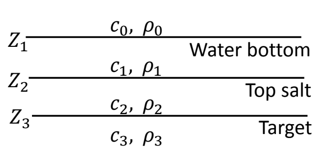

Figure 2: A 1D earth model, with three interfaces. The first interface, with depth Z_1 is the water bottom. The second interface, with depth Z_2, can be identified with the top salt and the third interface, with depth Z_3, can be identified with the target.

We consider data made of primaries and internal multiples created by spike-waves at normal incidence: $D(t) = R_1\delta(t-t_1) + R'_2\delta(t-t_2) + R'_3\delta(t-t_3) + IM$, with $R'_2 = T_{01}R_2T_{10}$, $R'_3 = T_{01}T_{12}R_2T_{21}T_{10}$. Also t_i is the travel time of the primary associated with the interface with depth Z_i, R_i is the reflection coefficient experienced by a wave when upward reflected at the interface with depth Z_i, and T_{ij} represents the transmission coefficient experienced by a wave traveling from the acoustic medium with parameters (c_i, ρ_i) to the acoustic medium with parameters (c_j, ρ_j).

In this case, the input of the leading-order attenuator, eq. (1), becomes:

$$b_1(z) = R_1\delta(z-z_1) + R'_2\delta(z-z_2) + R'_3\delta(z-z_3) + \cdots, \quad (2)$$

where $z_i = c_0t_i/2$ is the position of the reflector with depth Z_i, after Stolt's uncollapsed migration*. The z_i are usually referred to as *pseudodepths*, and we say that eq. (2) is in the *pseudodepth* domain.

Although the input data of the leading-order attenuator, eq. (2), includes primaries and internal multiples, we only consider the effect of the primaries. Initial steps towards the inclusion of internal multiples are addressed in Ma and Weglein (2012) and Liang and Weglein (2012). In the time domain the result for the evaluation of eq. (1), using eq. (2) is (See Weglein et al. 2003)

$$b_3(t) = -T_{01}T_{10} * (IM)_{j=1} + \cdots, \quad (3)$$

where $(IM)_{j=1}$ is the sum of the contributions to the data of all first-order internal multiples generated at the shallowest reflector of the model:

$$(IM)_{j=1} = -T_{01}R_2R_1R_2T_{10}\delta(t-(2t_2-t_1))$$
$$-2T_{01}R_2R_1T_{21}R_3T_{12}T_{10}\delta(t-(t_2+t_3-t_1))$$
$$-T_{01}T_{12}^2R_3R_1R_3T_{21}^2\delta(t-(2t_3-t_1)). \quad (4)$$

Consider now the contribution of the data and the leading-order attenuator $b_3(t)$ to the IMAS:

$$b_1(t) + b_3(t) = P + [1 - T_{01}T_{10}](IM)_{j=1} + \cdots, \quad (5)$$

where P stands for primaries. As $0 < T_{01}T_{10} < 1$, it follows from (5) that the amplitude contribution of $(IM)_{j=1}$ is reduced by an amount $T_{01}T_{10}$ with respect to their contribution previous to the addition of $b_3(t)$. $T_{01}T_{10}$ is referred as *attenuator factor*. An analogous situation is present for the internal multiple with downward reflection at the second reflector. However, in the present work we will only need the effects of $b_3(t)$ on $(IM)_{j=1}$.

THE ELIMINATION SUBSERIES

In the past section we showed, using the model of Figure 2, how the leading-order attenuator decreases the amplitude contribution for first-order internal multiples generated at the shallowest interface, by an amount of $T_{01}T_{10}$. This means that to promote this attenuation to an elimination, the contribution of higher-order terms from the elimination subseries need to

*For normal incidence of a spike-wave, the relation between $D(t)$ and $b_1(z)$ is as follows: 1) Fourier transform $D(t)$, 2) write the result, $D(\omega)$, in terms of $z_i = c_0t_i/2$ and the vertical wavenumber $k = 2\omega/c_0$ to end with a function $D(k)$ and 3) define $b_1(z) \equiv \mathscr{F}^{-1}[D(k)]$ where $\mathscr{F}^{-1}$ denotes the inverse Fourier transform

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move this attenuator factor to the unity: when those higher-order contributions are added to the initial attenuation provided by $b_3(t)$, the predicted amplitude will exactly match $(IM)_{j=1}$. Hence, the collective contribution of the terms in the elimination subseries will remove $(IM)_{j=1}$ from the data.

To isolate terms from the ISS with the right contributions, it is convenient to express 1 in terms of R_1. For this purpose the following geometric series expansion is useful:

$$1 = \frac{T_{01}T_{10}}{T_{01}T_{10}} = \frac{T_{01}T_{10}}{(1-R_1^2)} = T_{01}T_{10}(1+R_1^2+R_1^4+\cdots). \quad (6)$$

Notice that after distributing the product on the right hand side of eq. (6), the first term is the initial attenuation provided by the leading-order attenuator. Therefore, the remaining terms are the amplitude contribution required from higher-order terms, in any subseries claiming to promote the attenuation to elimination. We will focus in isolating the term with attenuation factor $T_{01}T_{10} * R_1^2$, the second term on the right hand side of eq. (6). We also want to predict the exact travel time of the internal multiples, i.e., we are looking for a term with contribution equal to $T_{01}T_{10} * R_1^2 * (IM)_{j=1}$.

Upon inspection of the ISS, we arrive to:

$$b_5^{(IM)_{j=1}}(k) \equiv \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'} F[b_1(z')] \times \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \quad (7)$$

$F[b_1(z')]$ is given by

$$F[b_1(z')] = \mathscr{F}^{-1}\left[\int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{z-\varepsilon}^{z+\varepsilon} dz_1 e^{-ikz_1} b_1(z_1) \times \int_{z_1-\varepsilon}^{z_1+\varepsilon} dz_2 e^{ikz_2} b_1(z_2)\right], \quad (8)$$

where $\mathscr{F}^{-1}$ means inverse Fourier transform, and the subindex in $b_5^{(IM)_{j=1}}$ means that it is of fifth-order in the data. The ε is applied in this context to include the self-interactions $z_2 = z_1$ and $z_1 = z$, rather than to avoid them, as is the case for the leading-order attenuator.

Upon evaluation of eqs. (7) and (8) using the primaries in eq. (2), the result in the time domain includes the expected contribution, plus additional terms which contribute to further attenuation of (presumably they also start the elimination of) other first-order internal multiples: $b_5^{(IM)_{j=1}}(t) = -T_{01}T_{10} * R_1^2 * (IM)_{j=1} + \cdots$.

Consider now the sum of the data, the leading-order attenuator and $b_5^{(IM)_{j=1}}$:

$$b_1(t) + b_3(t) + b_5^{(IM)_{j=1}}(t) = P + [1 - T_{01}T_{10}(1+R_1^2)](IM)_{j=1} + \cdots. \quad (9)$$

Eq. (9) makes evident that in this case the attenuation factor $T_{01}T_{10}$ is changed to $T_{01}T_{10}(1+R_1^2)$. This attenuation contains the first and second terms of the geometric series on the right hand side of eq. (6). Hence, the expression proposed for $b_5^{(IM)_{j=1}}$ in eqs. (7) and (8) correctly reproduces the required amplitude contribution to move the attenuation of $(IM)_{j=1}$ a step closer to elimination.

Higher-order contributions for the elimination subseries are analogous to eq. (7) but with an appropriate $F[b_1(z')]$, e.g. the function $F[b_1(z')]$ for the term following $b_5^{(IM)_{j=1}}$, denoted $b_7^{(IM)_{j=1}}$, and with contribution $T_{01}T_{10} * R_1^4 * (IM)_{j=1}$ is

$$F[b_1(z')] = \mathscr{F}^{-1}\left[\int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{z-\varepsilon}^{z+\varepsilon} dz_1 e^{-ikz_1} b_1(z_1) \times \int_{z_1-\varepsilon}^{z_1+\varepsilon} dz_2 e^{ikz_2} b_1(z_2) \int_{z_2-\varepsilon}^{z_2+\varepsilon} dz_3 e^{-ikz_3} b_1(z_3) \times \int_{z_3-\varepsilon}^{z_3+\varepsilon} dz_4 e^{ikz_4} b_1(z_4)\right]. \quad (10)$$

Following this line of thinking, further contributions to the elimination of $(IM)_{j=1}$ can be isolated to get the elimination subseries:

$$b^{(IM)_{j=1}}(t) = b_1(t) + b_3(t) + b_5^{(IM)_{j=1}}(t) + b_7^{(IM)_{j=1}}(t) + \cdots. \quad (11)$$

We can use as many terms as we need, in order to achieve a desired degree of accuracy in the prediction of an internal multiple (of first-order and generated at the shallowest reflector).

APPLICATION OF THE ELIMINATION SUBSERIES TO AN ANALYTIC MODEL

In this section we will use an analytic model in which an internal multiple of first-order is interfering destructively with a primary. This is to show the usefulness of the eliminator subseries by surgically removing the internal multiple.

The analytic model we will focus is the three-interface model of Figure 2, with specific values for the acoustic parameters assigned as $(1500m/s, 1000kg/m^3)$, $(2280m/s, 1000kg/m^3)$, $(9000m/s, 1700kg/m^3)$ and $(9900, 1578kg/m^3)$ for (c_0, ρ_0), (c_1, ρ_1), (c_2, ρ_2) and (c_3, ρ_3) respectively. The Primary created at the interface with depth Z_i is denoted P_i. First-order internal multiples are denoted as IM_{ijk} with j indicating the reflector in which the downward reflection is generated; i and k indicate the reflectors where the first and second upward reflections are generated respectively.

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The interfering events are the primary P_3 and the internal multiple IM_{212}, whose common travel time is 2.2947s. The amplitudes for P_3 and IM_{212} are 0.0045 and -0.1084 respectively. A trace is shown in Figure 3, from which the amplitude of the combined event $P_3 + IM_{212}$ can be read as -0.1039: the polarity is opposite to that of the primary.

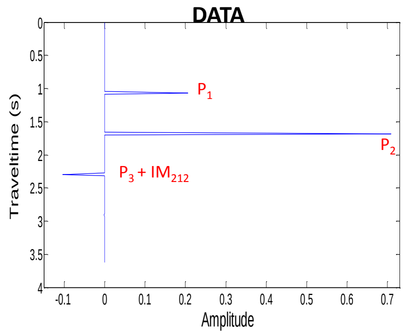

Figure 3: Data of the model. This data includes primaries and the relevant internal multiples of first order.

Next is the application of $b_3(t)$ to attenuate internal multiples of first-order. For the interfering event the amplitude after the action of $b_3(t)$ is -0.0001 and hence the amplitude attenuation is not enough to change the polarity of the interfering event. This might lead to assign to the primary an incorrect polarity.

From the above paragraph it is evident that improvement in the predicted amplitude for IM_{212} is necessary. This is possible if we include further terms from the elimination subseries isolated in the previous section. This is shown in Figure 4, in which the effect of the third term, $b_5^{(IM)_{j=1}}(t)$, has been included in addition to $b_3(t)$.

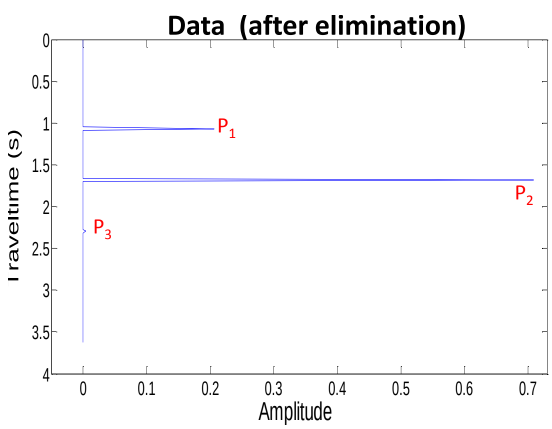

Figure 4: Data after the action of both, the leading-order attenuator and $b_5^{(IM)_{j=1}}(t)$.

The primary P_3 is now with its original amplitude and polarity, 0.0045, which means that the interfering internal multiple has been removed. However, for more complex models the convergence can be slower, and more terms might be needed. Also, from Figure 4, it can be noticed that neither the travel times nor the amplitudes of the primaries P_1 and P_2 are changed, as expected from a method for surgical removal of internal multiples.

DISCUSSION AND CONCLUSIONS

We have isolated from the ISS a subseries whose task is to eliminate first-order internal multiples generated at the shallowest interface, and also attenuates internal multiples from all deeper reflectors. This elimination subseries predicts the phase and the exact amplitude of the internal multiples and does not modify any primary. Therefore, the surgical removal of such internal multiples is achieved.

We have also applied the eliminator subseries to an analytic example with three interfaces. The configuration is set up to produce an internal multiple (with downward reflection at the shallowest reflector) interfering destructively with the primary generated at the third reflector, in a way that the leading-order attenuator is not enough to let the primary show up in the data with its correct polarity. We show how the action of the third-order and fifth-order contributions of the algorithm remove the interfering internal multiple, making the primary to appear in the trace with its original amplitude and polarity. In practice however, it is not possible to know a priori the number of terms that are necessary to eliminate the interfering internal multiple. The recipe is to apply to the data one term at a time until no change is noticed in the primary. Although higher-order terms will imply an increased computational cost (more integrals need to be calculated), if the interfering primary is suspected to be the target, then the investment might be worthwhile, as a situation involving a drilling or no drilling decision might be involved and processing costs pale compared to drilling dry holes.

Interfering events are common in onshore exploration, but they may also occur offshore. Therefore, the algorithm in this work may provide added value in those challenging geologic configurations in which techniques such as the energy-minimization adaptive subtraction fails.

Further research in this topic includes extending the method beyond the normal incidence assumption of the present work, and to derive the corresponding multidimensional version of the subseries presented here. Additionally, current challenges in exploration seismology might also require the removal of other internal multiples of first-order, generated beneath the shallowest reflector. Hence, a more generalresearch goal is to isolate a subseries, with the specific task of the elimination of first-order internal multiples generated at all reflectors.

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EDITED REFERENCES

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